DAMAGE MECHANISM IN THE CERMET FUEL ROD SHELL UNDERGOING RAPID COOLING: ANALYTICAL MODELING

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ABSTRACT

Destruction of the fuel rod shells in nuclear reactors lead to hazardous emergency situations. There can be various causes and scenarios of such accidents. One of the reasons can be an overcooling of the circulating medium surrounding the fuel rod. We consider a problem of calculation of the stress-strain condition in the cylindrical and spherical type fuel rod shells undergoing a rapid cooling of the outer surface [1, 2].
The outer surface of the shell is cooled down to a reference value of 0 °C. We analyse the resulting process of development of the plastic deformation zone in the material of the shell.
The transient problem is solved for the heat transfer calculation. The stress-strain condition is calculated using a quasi-stationary approach in which the wave effects in the material are assumed as negligible.

Key words: transient thermal analysis, nuclear reactor, cylindrical and spherical fuel rod, fuel rod shell

1. ANALYTICAL MODEL FOR THE CYLINDRICAL SHELL

As a first approximation, we consider the solution of a non-stationary thermoelastic problem for a finite hollow circular cylinder. Let the cylinder of length L has temperature $T = T_0$ when t < 0. At time t = 0, the medium temperature at the inner and outer surfaces of the cylinder varies according to predefined laws, but the temperature field remains axisymmetric. At the ends, a heat exchange with the medium whose temperature is equal to T_0 also takes place.
In a cylindrical coordinate system (*r*, φ, *z*) for the axisymmetric problem $\sigma_{r\varphi} = \sigma_{z\varphi} = 0$. The components of displacements are *u* in radial, and *w* in axial directions. They don't depend on φ. The equations of motion in the cylindrical coordinate system are

$$\begin{aligned} &\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{1}{r}\left(\sigma_{rr} - \sigma_{\varphi\varphi}\right) = \rho \frac{\partial^2 u}{\partial t^2}, \\ &\frac{\partial \sigma_{rz}}{\partial z} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{z} = \rho \frac{\partial^2 w}{\partial t^2}. \end{aligned} \tag{1.1}$$

Cauchy ratio is represented as

$$\begin{aligned} &e_{rr} = \frac{\partial u}{\partial r}, \quad e_{\varphi\varphi} = \frac{u}{r}, \quad e_{zz} = \frac{\partial w}{\partial z}, \\ &e_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right), \quad e = \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z}. \end{aligned} \tag{1.2}$$

Hooke's law in the cylindrical coordinate system

$$
\begin{aligned}
&\sigma_{rr} = 2\mu e_{rr} + \lambda e + \alpha T, \quad \sigma_{r\varphi} = 2\mu e_{r\varphi}, \\
&\sigma_{\varphi\varphi} = 2\mu e_{\varphi\varphi} + \lambda e + \alpha T, \quad \sigma_{rz} = 2\mu e_{rz}, \\
&\sigma_{zz} = 2\mu e_{zz} + \lambda e + \alpha T, \quad \sigma_{\varphi z} = 2\mu e_{\varphi z},
\end{aligned} \tag{1.3}
$$

where λ, μ are Lamé constants. Substituting (1.3) into (1.1), considering (1.2), we obtain the equations for the displacements *u* and *w*

$$
\begin{aligned}
&\Delta u - \frac{u}{r^2} + \frac{1}{1-2\mu}\frac{\partial e}{\partial r} - \frac{\rho}{G}\frac{\partial^2 u}{\partial t^2} = \frac{2(1+\mu)}{1-2\mu}\frac{\partial(\alpha T)}{\partial r}, \\
&\Delta w + \frac{1}{1-2\mu}\frac{\partial e}{\partial z} - \frac{\rho}{G}\frac{\partial^2 w}{\partial t^2} = \frac{2(1+\mu)}{1-2\mu}\frac{\partial(\alpha T)}{\partial z},
\end{aligned} \tag{1.4}
$$

where the Laplace operator Δ is written in the cylindrical coordinate system:

$$
\Delta = \frac{\partial}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{\partial^2}{\partial z^2} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}.
$$

Heat equation

$$
\frac{\partial T}{\partial t} = a\Delta T, \tag{1.5}
$$

is solved under the conditions:

- *initial conditions*

$$
T = T_0 \text{ at } t = 0. \tag{1.6}
$$

- *boundary conditions*

$$
a_{11}T + a_{12}\frac{\partial T}{\partial z} = \theta_{(l)}(r,\ t) \text{ when } z = l, \tag{1.7}
$$

$$
b_{11}T + b_{12}\frac{\partial T}{\partial z} = \theta_{(0)}(r,\ t) \text{ when } z = 0, \tag{1.8}
$$

$$
c_{11}T + c_{12}\frac{\partial T}{\partial z} = \varphi_{(e)}(z,\ t) \text{ when } r = R_{(e)}, \tag{1.9}
$$

$$
d_{11}T + d_{12}\frac{\partial T}{\partial z} = \varphi_{(i)}(z,\ t) \text{ when } r = R_{(i)}. \tag{1.10}
$$

We assume that the outer and inner surfaces of the cylinder are free from stresses. Let's, at first, consider the determination of the temperature field. For this purpose we divide problem (1.5) - (1.10) into two. To simplify, we suppose that $T_0 = 0$ in (1.6) and the solution of *T* is presented as a sum

$$
T = T_1 + T_2. \tag{1.11}
$$

For T_1 we solve equation (1.5), assuming that the following conditions are met

$$a_{11}T_1 + a_{12}\frac{\partial T_1}{\partial z} = 0 \text{ when } z = l, \tag{1.12}$$

$$u_{11}T_1 + u_{12}\frac{\partial T_1}{\partial z} = 0 \text{ when } z = 0, \tag{1.13}$$

$$c_{11}T_1 + c_{12}T_1 = \varphi_{(e)}(z,\ t) \text{ when } r = R_{(e)}, \tag{1.14}$$

$$d_{11}T_1 + d_{12}T_1 = \varphi_{(i)}(z,\ t) \text{ when } r = R_{(i)}, \tag{1.15}$$

and for T_2 we solve equation (1.5), assuming that the following conditions take place

$$a_{11}T_2 + a_{12}\frac{\partial T_2}{\partial z} = \theta_{(e)}(r,t) \text{ when } z = l, \tag{1.16}$$

$$b_{11}T_2 + b_{12}\frac{\partial T_2}{\partial z} = \theta_{(i)}(r,t) \text{ when } z = 0, \tag{1.17}$$

$$c_{11}T_2 + b_{12}\frac{\partial T_2}{\partial r} = 0 \text{ when } r = R_{(e)}, \tag{1.18}$$

$$b_{11}T_2 + b_{12}\frac{\partial T_2}{\partial r} = 0 \text{ when } r = R_{(i)}. \tag{1.19}$$

2. FORMALISM FOR A SPHERICAL SHELL

A sphere with an outer radius $r = R_2$ and the inner $r = R_1$ is under the influence of temperature field T_2 when $r = R_2$ T_1 on the inner surface when $r = R_1$. We introduce thermoelastic potential Φ for displacements

$$u_i = \frac{\partial \Phi}{\partial i}, \quad (i = x,\ y,\ z). \tag{2.1}$$

The equations for Φ in Cartesian coordinates are obtained from the thermal conductivity equations in terms of displacements by substituting expression (2.1) into them and at certain transformations

$$\Delta\Phi - \frac{1-2\mu}{2(1-\mu)}\frac{\rho}{G}\frac{\partial^2\Phi}{\partial t^2} = \frac{1+\mu}{1-\mu}\alpha T, \tag{2.2}$$

where μ is Poisson's ratio, α is the coefficient of thermal conductivity, ρ is the density, G is the shear modulus. The substitution of (2.1) into Cauchy conditions and then into Hooke's law gives the formulae for stresses through potential Φ.

$$\sigma_{ik} = 2G\left\{\frac{\partial^2\Phi}{\partial i\partial k} + \frac{\delta_{ik}}{1-2\mu}\left[\mu\Delta\Phi - (1+\mu)\alpha T\right]\right\}. \tag{2.3}$$

Let's exclude from (2.3) the temperature factor $(1+\mu)\alpha T$ using (2.2), then we obtain

$$\sigma_{ik} = 2G\left(\frac{\partial^2\Phi}{\partial i\partial k} - \Delta\Phi\,\delta_{ik}\right) + \rho\frac{\partial^2\Phi}{\partial t^2}\delta_{ik}. \tag{2.4}$$

In spherical coordinates, in the case of central symmetry we have from (2.3)

$$\sigma_{rr} = -\frac{4G}{r}\frac{\partial \Phi}{\partial r} + \rho\frac{\partial^2 \Phi}{\partial t^2},$$
$$\sigma_{\varphi\varphi} = \sigma_{\theta\theta} = 2G\left(\frac{1}{r}\frac{\partial \Phi}{\partial r} - \Delta\Phi\right) + \rho\frac{\partial^2 \Phi}{\partial t^2}. \tag{2.5}$$

The equation for the displacement in this case is

$$\Delta u - \frac{2}{r^2 - 2(1-\mu)}\frac{1-2\mu}{G}\frac{\rho}{}\frac{\partial^2 u}{\partial t^2} = \frac{1+\mu}{1-\mu}\frac{\partial(\alpha T)}{\partial r}. \tag{2.6}$$

In case of slow temperature change, so that Duhamel's hypothesis can be used, the equation of potential displacement in the spherical coordinates becomes

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial \Phi}{\partial r}\right) = \frac{1+\mu}{1-\mu}ar^2T. \tag{2.7}$$

In equation (2.7) we neglect the inertial forces. Integrating (2.7) over r we obtain

$$\Phi = \frac{1+\mu}{1-\mu}\frac{\alpha}{3}\int_{R_2}^{r}\xi(T(\xi,\ t))d\xi. \tag{2.8}$$

The average temperature of the sphere at time t, depending on r, is

$$\langle T(t)\rangle = \frac{3}{r^3}\int_{R_2}^{r}\xi^2T(t)d\xi. \tag{2.9}$$

Let’s consider the stationary case $T(r,\ t) = T(r)$. The heat equation can be written as

$$\frac{\partial^2(r,\ T_s)}{\partial r^2} = 0,\ T_s = T_1 \text{ when } r = R_1, \text{ and } T_s = T_2 \text{ when } r = R_2.$$

By means of direct integration we obtain

$$T_s = c + \frac{c_1}{r},\ c = \frac{T_1R_1 - T_2R_2}{R_1 - R_2},\ c_1 = \frac{(T_2 - T_1)R_1R_2}{R_1 - R_2}. \tag{2.10}$$

The heat equation in the spherical coordinates is

$$\frac{\partial T}{\partial t} = \frac{a}{r}\frac{\partial^2(rT)}{\partial r^2}. \tag{2.11}$$

At $t = 0$ and $r = R_2$ the temperature decrease appears stepwise from T_2 to 0, whereas at $t > 0$ and $r = R_2$ zero temperature is set. Then

$$T\,(r,\,0) = T_s\,(r), \quad T\,(R_1,\,t) = T_1, \quad T\,(R_2,\,t) = 0. \tag{2.12}$$

Equation (2.11) is solved by the method of separation of variables

$$T(r,\ t) = T_{(r)}^{(r)} T_{(t)}^{(t)}. \tag{2.13}$$

Considering (2.13) we obtain from (2.11)

$$\frac{a}{r}\frac{d^2\left(rT^{(t)}\right)}{T^{(r)}dr^2} = \frac{dT^{(t)}}{T^{(t)}dt} = const = A\,. \tag{2.14}$$

$$T_{(t)}^{(r)} = e^{At+C} = Ce^{At}, \qquad \frac{A}{d} = -\lambda^2 \tag{2.15}$$

$$\frac{d^2\left(rT^{(r)}\right)}{rdr^2} = +\lambda T^{(r)} = 0 \tag{2.16}$$

Solution (2.16) can be rewritten as

$$T_{(r)}^{(r)} = \frac{1}{r}\left[C_1(\lambda)\cos\lambda r + C_2(\lambda)\sin\lambda r\right]. \tag{2.17}$$

Thus a particular solution of equation (2.11) for any λ is

$$T_{(r,\,t)}^{(\lambda)} = \left(C_1(\lambda)\frac{\cos\lambda r}{r} + C_2(\lambda)\frac{\sin\lambda r}{r}\right)\exp\left(-\lambda^2 t\right). \tag{2.18}$$

And we present the general solution as

$$T_{(r,\,t)} = \int_{\partial}^{\infty}\left[C_1(\lambda)\frac{\cos\lambda r}{r} + C_2(\lambda)\frac{\sin\lambda r}{r}\right]\exp\left(-\lambda^2 t\right)d\lambda\,. \tag{2.19}$$

3. CONCLUSION

The proposed analytical method allows us to solve the problem of the stress distribution in cylindrical and spherical shells in case of a temperature shock. The relations obtained can be used to investigate problematic zones and residual stresses in water cooled fuel rods.

4. REFERENCES

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